APPENDIX 3: PRINCIPLES OF DYNAMIC ANALYSIS

APPENDIX 3A ILLUSTRATIVE EXAMPLES

PROBLEM 3A-1 ULTIMATE UNIT RESISTANCE

Problem: **Determine the ultimate unit resistance of a two-way structural element using (1) general solution and (2) charts.**

Procedure: **Part (a) - General Solution**

Step 1. Establish design parameters.

Step 2. Assume yield line locations in terms of *x* and/or *y* considering support conditions, presence of openings, etc.

Step 3. Determine negative and positive moment capacities of sections crossed by assumed yield lines.

Step 4. Establish distribution of moments across negative and assumed yield lines, considering corner effects and those of openings.

Step 5. Determine the ultimate unit resistance for each sector in terms of *x* and/or *y* considering free body diagram of the sectors (Figure 3-3 (+)). Summation of the moments about the axis of rotation (support) of the sector yields Equation 3-3.

Step 6. Equate the ultimate unit resistance of the sectors and solve for the yield line location *x* and/or *y*.

Step 7. With known yield line location, solve for ultimate unit resistance of the element, using equations obtained in Step 6.

Note: For complex problems (three or more different sectors) the solution for the ultimate unit resistance is most easily accomplished through a trial-and-error procedure by determining r_u for each sector for a given (assumed) yield line location and adjusting the yield lines until the several values of r_u agree to within a few percent.

Procedure: **Part (b) - Chart Solution**

Step 1. Same as in Step 1 of Part (a).

Step 2. Same as in Step 2 of Part (a).

Step 3. Determine the negative and positive ultimate moment capacities in vertical and horizontal directions.

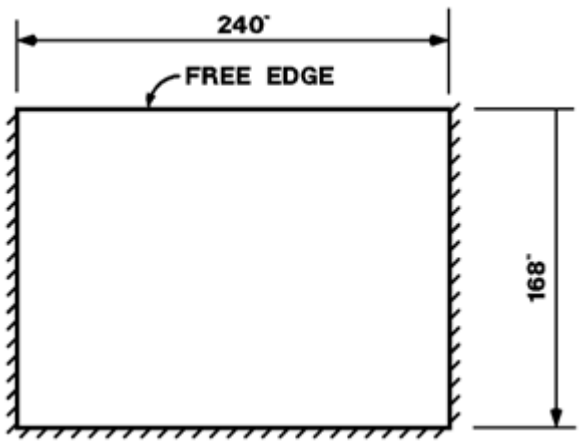
Step 4. For given support conditions (and value of x_2/x_1 in the case of an element with three edges supported and fourth free), use the appropriate chart (Figures 3-4 (+) through 3-20 (+)) to obtain yield line location ratios x/L or y/H for value of quantity obtained in Step 4. Then calculate *x* or *y*.

Step 5. Using the appropriate equation from Table 3-2 (+) determine the ultimate unit resistance of the element.

EXAMPLE 3A-1 (A) ULTIMATE RESISTANCE

Required: **Ultimate unit resistance of two-way structural steel element shown below using (1) general solution and (2) charts.**

Figure 3A-1



Solution: **Part (a) - General Solution**

Step 1. Given:

(a) L = 240 in H = 168 in

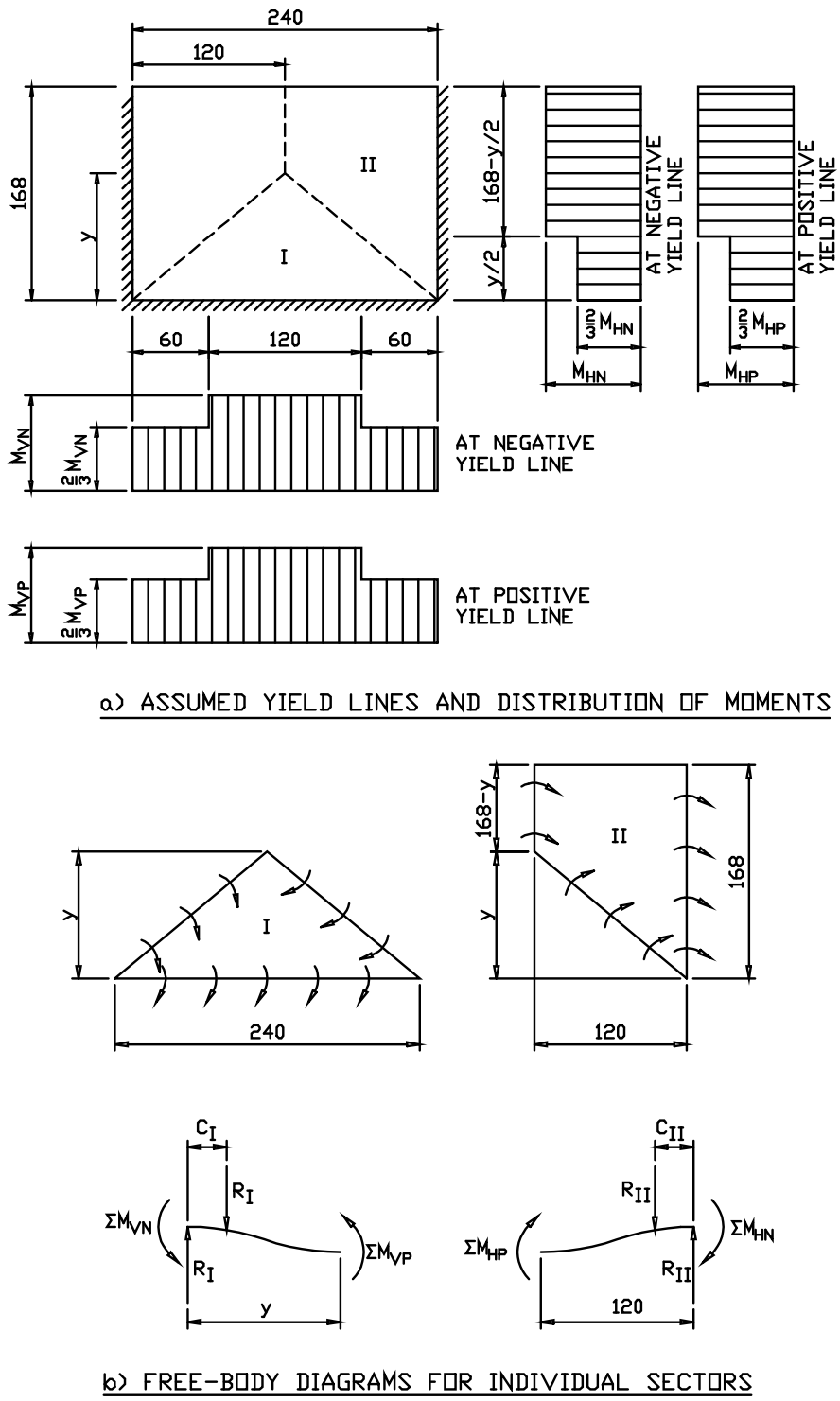
(b) Fixed on three sides and free at the fourth

Step 2. Assume yield line location (Figure 3A-2 (+))

Step 3. The negative and positive moment capacities in both the horizontal and vertical directions are determined from the properties of the material. For this Example, it will be assumed that the moment capacities are equal to M = 20,000 in·lbs/in.

$M_{HN} = M_{HP} = M_{VN} = M_{VP}$ = 20,000 in·lbs/in

Figure 3A-2


240
120
168
y
I
II
168-y/2
y/2
AT NEGATIVE YIELD LINE
AT POSITIVE YIELD LINE
60
120
60
M_{VN}
$\frac{2}{3}M_{VN}$
M_{VP}
$\frac{2}{3}M_{VP}$
$\frac{2}{3}M_{HN}$
M_{HN}
$\frac{2}{3}M_{HP}$
M_{HP}
a) ASSUMED YIELD LINES AND DISTRIBUTION OF MOMENTS
168-y
168
240
C_I
R_I
ΣM_{VN}
ΣM_{VP}
C_{II}
R_{II}
ΣM_{HP}
ΣM_{HN}
b) FREE-BODY DIAGRAMS FOR INDIVIDUAL SECTORS

Step 4. For distribution of moments across negative and assumed positive lines, see Figure 3A-2(a) (+).

Step 5. The ultimate unit resistance of each sector is obtained by taking the summation of the moments about its axis of rotation (supports) so that $\Sigma M_N + \Sigma M_P = Rc = r_u Ac$

a. Sector I (Figure 3A-2 (+))

$$\Sigma M_{VN} + \Sigma M_{VP} = 120(20{,}000) + 2(2/3)(20{,}000)(60) + 120(20{,}000) + 2(2/3)(20{,}000)(60) = 8.0\text{x}10^6 \text{ in-lbs.}$$

$$r_u Ac = r_u\left[\frac{240(y)}{2}\right]\left[\frac{y}{3}\right] = 40 r_u y^2$$

therefore,

$$r_u = 400(20{,}000)/40y^2 = 0.2\text{x}10^6 / y^2$$

b. Sector II (Figure 3A-2 (+))

$$\Sigma M_{HN} + \Sigma M_{HP} = (168\text{-}y/2)(20{,}000) + (2/3)(20{,}000)(y/2) + (20{,}000)(168\text{-}y/2) + (2/3)(20{,}000)(y/2) = 336(20{,}000) - (y/3)(20{,}000)$$

$$r_u Ac = r_u\left[\frac{120(168+168-y)}{2}\right]\left[\frac{120(168+2(168-y))}{3}\right]/(168+168-y) = 4{,}800 r_u (252-y)$$

therefore,

$$r_u = \frac{336(20{,}000) - \frac{y}{3}(20{,}000)}{4800(252-y)}$$

Step 6. Equate the ultimate unit resistance of the sectors.

$$\frac{10(20{,}000)}{y^2} = \frac{336(20{,}000) - \frac{y}{3}(20{,}000)}{4800(252-y)}$$

Simplifying:

$y^3 - 1008y^2 - 144000y + 36288000 = 0$

and the desired root is: $y = 137.6$ in.

Step 7. The ultimate unit resistance is obtained by substituting the value of y into either equation obtained in Step 5, both of which yield:

$r_u = (20{,}000)/(137.6)^2 = 10.6$ psi

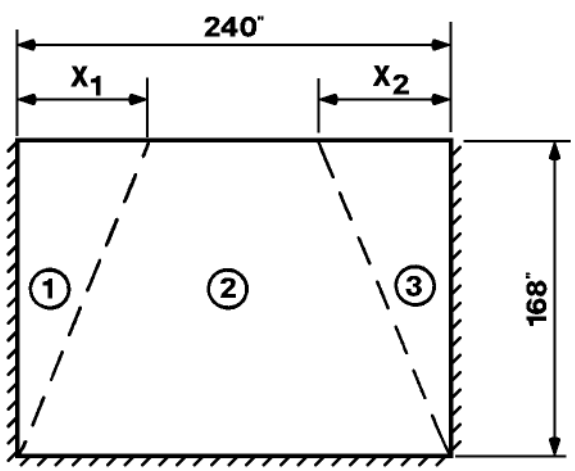
Solution: Part (b) - Chart Solution

Note: Element conforms to the requirements of Section 3-8 since it is fixed on three sides and free on the remaining side and has uniform thickness in the horizontal and vertical directions.

Step 1. Same as Step 1 in Part (a).

Step 2. For illustrative purposes, a different yield pattern (Figure 3A-3 (+)) will be assumed.

Figure 3A-3



Step 3. For ultimate moment capacities, see Step 3 of Part (a).

Step 4. For three sides fixed and the fourth free, calculate the parameter.

$X_2/X_1=[(M_{HN3} + M_{HP})/(M_{HN1} + M_{HP})]^{1/2} = [(20{,}000)(2)/(20{,}000)(2)]^{1/2} = 1.0$

From Figure 3-11 (+), ($X_2/X_1 = 1.0$) calculate the parameters:

$L/H[M_{VP}/(M_{HN1} + M_{HP})] = (240/168)[20{,}000/(2)(20{,}000)]^{1/2} = 1.01$

and

$$\frac{M_{VP}}{M_{VN2}} = \frac{20{,}000}{20{,}000} = 1.0$$

Read yield line location:

X_1/L exceeds the maximum possible value of 0.5 therefore, assumed yield line pattern is wrong. Assume alternate yield line pattern as shown in Figure 3A-2 (+).

From Figure 3-16 (+) calculate the following parameters:

$$\frac{L}{H} = \frac{(M_{VN3} + M_{VP})^{1/2}}{(M_{HN2} + M_{HP})^{1/2} + (M_{HN1} + M_{HP})^{1/2}}$$

$$= \frac{240}{168}\left[\frac{(20{,}000 + 20{,}000)^{1/2}}{(20{,}000 + 20{,}000)^{1/2} + (20{,}000 + 20{,}000)^{1/2}}\right] = 0.71$$

and

$$\frac{X}{L} = \frac{[(M_{HN1} + M_{HP})/(M_{HN2} + M_{HP})]^{1/2}}{1 + [(M_{HN1} + M_{HP})/(M_{HN2} + M_{HP})]^{1/2}}$$

$$= \frac{[40{,}000/40{,}000]^{1/2}}{1 + [40{,}000/40{,}000]^{1/2}} = 1/2$$

From Figure 3-16 (+) read of yield line location:

y/H = 0.82; y = 0.8(168) = 137.6 in

X/L = 0.50; X = 0.5(240) = 120.0 in

Step 5. From Table 3-2 (+):

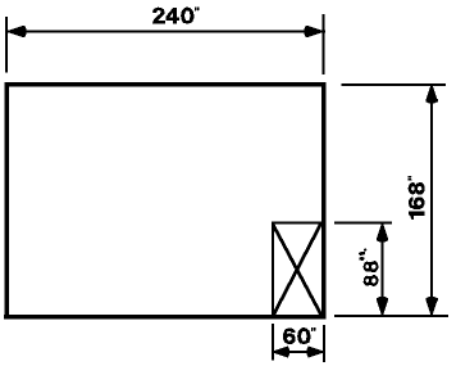
NOTE: Both equations given in the table for each edge condition and yield line location will provide identical values of r_u.

$$r_u = \frac{5(M_{VN} + M_{VP})}{y^2} = \frac{5(20{,}000 + 20{,}000)}{137.6^2} = (0.00055)20{,}000 = 10.5 \text{ psi}$$

EXAMPLE 3A-1 (B) ULTIMATE UNIT RESISTANCE

Required: **Ultimate unit resistance of the element considered in Example 3A-1(A) except there is an opening as shown in Figure 3A-4 (+).**

Figure 3A-4



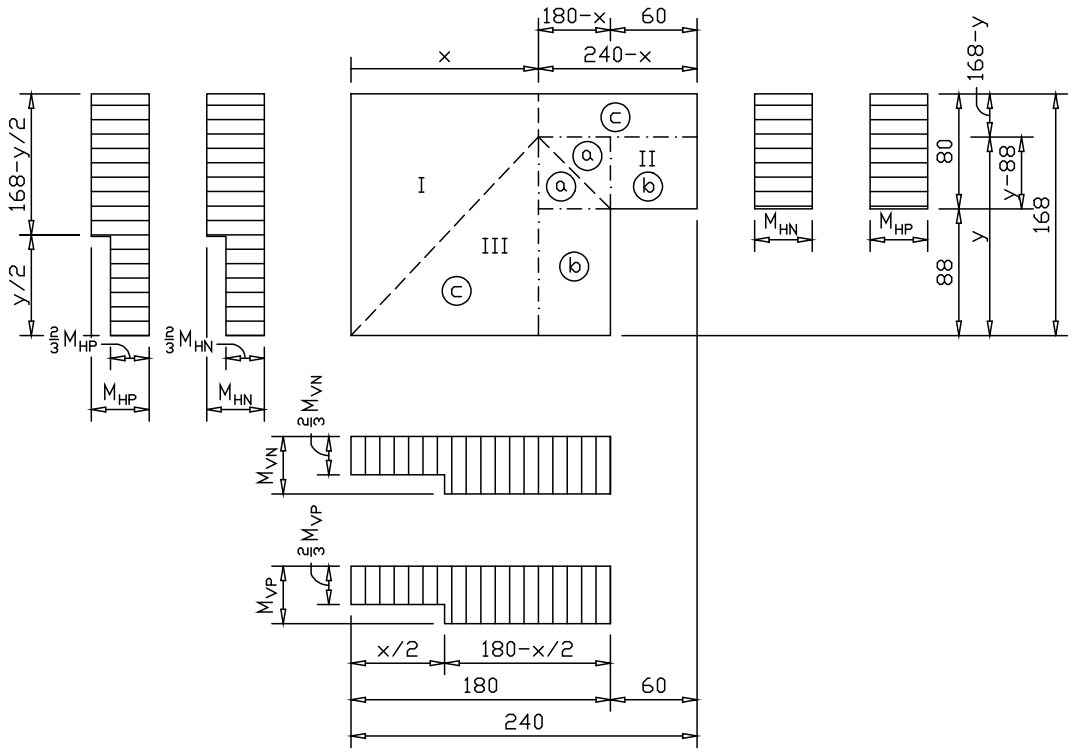
Solution:

Step 1. Given: (a) L = 240" H = 168" Two additional free edges are formed due to the presence of the opening.

Step 2. Assumed yield line location is shown in Figure 3A-5 (+) (three different sectors are formed).

Step 3. Same as Step 3 of Example 3A-1(A), Part (a).

Figure 3A-5



Step 4. For distribution of moments across negative and assumed positive yield lines see Figure 3A-5 (+). (Since opening is located at lower right corner, there is no reduced moment capacity in this area.)

Step 5. The ultimate unit resistance is obtained from:

$$\Sigma M_N + \Sigma M_p = Rc = r_u Ac$$

a. Sector 1 (Figure 3A-5 (+))

$$\Sigma M_{HN} + \Sigma M_{HP} = (20{,}000)(168\text{-}y/2) + (2/3)(20{,}000)(y/2) + (20{,}000)(168\text{-}y/2) + (2/3)(20{,}000)(y/2)$$

$$= 336(20{,}000) - (y/3)(20{,}000)$$

$$= (336\text{-}y/3)(20{,}000)$$

$$r_u Ac = r_u[x(168+168\text{-}y)/2]\,[x(168+2(168\text{-}y))/3(168+168\text{-}y)]$$

$$= r_u x^2(252\text{-}y)/3$$

therefore,

$$r_u = \frac{(1008 - y)(20{,}000)}{x^2(252 - y)}$$

b. Sector II (Figure 3A-5 (+))

$\Sigma M_{HN} + \Sigma M_{HP}$ = (20,000)(80) + (20,000)(80)

= 160(20,000)

Note: The sector is divided into Parts a, b, and c so that the centroid may be obtained (see table below).

Portion of Sector	Area (*A'*)	Distance from Centroid to axis of rotation (*c'*)	*A'c'*
a	$\frac{(y-88)(180-x)}{2}$	$\frac{(180-x)}{2}+60=\frac{360-x}{3}$	$\frac{(y-88)(180-x)(360-x)}{6}$
b	$(y-88)(60)$	$\frac{60}{2}$	$\frac{(y-88)(60)^2}{2}$
c	$(168-y)(240-x)$	$\frac{(240-x)}{2}$	$\frac{(168-y)(240-x)^2}{2}$

$$Ac=\sum A'c'=\frac{(y-88)(180-x)(360-x)}{6}+\frac{(y-88)(60)^2}{2}+\frac{(168-y)(240-x)^2}{2}$$
$$=\frac{1}{2}\left[\frac{(y-88)}{3}\left[(180-x)(360-x)+10800\right]+(168-y)(240-x)^2\right]$$

$$r_u=\frac{\left(\sum M_{HN}+\sum M_{HP}\right)}{Ac}$$
$$=\frac{6{,}400{,}000}{\frac{(y-88)}{3}\left[(180-x)(360-x)+10{,}800\right]+(168-y)(240-x)^2}$$

c. Sector III (Figure 3A-5 (+))

$\Sigma M_{vn} + \Sigma M_{vp}$ = (20,000)(180-*x*/2) + (2/3)(20,000)(*x*/2) +

(20,000)(180-*x*/2) + (2/3)(20,000)(*x*/2)

= 360(20,000) - (20,000)(*x*/3)

Portion of Sector	Area (*A'*)	Distance from Centroid to axis of rotation (*c'*)	*A'c'*
a	$\frac{(108-x)(y-88)}{2}$	$\frac{(y-88)}{3}+88=\frac{y+176}{3}$	$\frac{(180-x)(y-88)(y+176)}{6}$
b	$(180-\mathrm{x})(88)$	$\frac{88}{2}$	$\frac{(180-x)(88)^2}{2}$
c	$\frac{xy}{2}$	$\frac{y}{3}$	$\frac{xy^2}{6}$

$$Ac = \Sigma A'c' = (y\text{-}88)(180\text{-}x)(y+176)/6 + (180\text{-x})(88)^2/2 + xy^2/6$$

$$= 1/6\ (180\text{-}x)[(y\text{-}88)(y+176)+23{,}232] + xy^2$$

$$r_u = \frac{\sum M_{HN} + \sum M_{HP}}{Ac}$$

$$= \frac{(2160-2x)(20{,}000)}{(180-x)[(y-88)(y+176)+23{,}232]+xy^2}$$

Step 6. Due to the complexity of obtaining a direct solution for ultimate unit resistance, a trial-and-error solution will be used (see table below):

x	*y*	r_I	r_{II}	r_{III}
125	130	9.21	7.67	9.33
125	135	9.55	7.92	8.77
125	140	9.92	8.19	8.25
125	145	10.32	8.48	7.78
125	150	10.77	8.79	7.35
130	130	8.52	8.29	9.50
130	135	8.83	8.55	8.92
131	135	8.70	8.68	8.85

Therefore:

x = 131 in

y = 135 in

r_u = 8.68 psi

PROBLEM 3A-2 RESISTANCE-DEFLECTION FUNCTION

Problem: **Determine the actual and equivalent resistance deflection function in the elasto-plastic region for a two-way structural element.**

Procedure:

Step 1. Establish design parameters.

a. Geometry of element

b. Support conditions

Step 2. Determine ultimate positive and negative moment capacities.

Step 3. Determine static properties:

a. Modules of elasticity for the element

b. Moment of inertia of the element

Step 4. Establish points of interest and their ultimate moment capacities (Figure 3-23 (+))

Step 5. Compute properties at first yield.

a. Location of first yield

b. Resistance at first yield r_e

c. Moments at remaining points consistent with r_e

d. Maximum deflection at first yield

Step 6. Compute properties at second yield.

a. Remaining moment capacity at other points

b. Location of second yield

c. Change in unit resistance Δr between first and second yield.

d. Unit resistance at second yield r_{ep}

e. Moment at remaining point consistent with r_{ep}

f. Change in maximum deflection

g. Total maximum deflection

Note: An element with unsymmetrical support conditions may exhibit three or four support yields. Therefore, repeat Step 6 as many times as necessary to obtain properties at the various yield points.

Step 7. Compute properties at final yield (ultimate unit resistance):

a. Ultimate unit resistance

b. Change in resistance between ultimate unit resistance and resistance at prior yield

c. Change in maximum deflection (for elements supported on two, three, or four sides, use stiffness obtained from Figure 3-26 (+), 3-30 (+) and 3-36 (+), respectively)

d. Total maximum deflection

Step 8. Draw the actual resistance-deflection curve. (Figure 3-39 (+))

Step 9. Calculate equivalent maximum elastic deflection of the element.

EXAMPLE 3A-2 RESISTANCE-DEFLECTION FUNCTION

Required: **The actual and equivalent resistance-deflection function (curve) in the elasto-plastic region for the two-way structural steel element.**

Solution:

Step 1. Given:

a. L = 240 in H = 168 in

b. Fixed on three sides and free at the fourth

Step 2. Same as Step 3 of Example 3A-1(A), Part a.

Step 3. Static properties:

a. Modulus of elasticity, E_s for steel

$E_s = 29 \times 10^6$ psi

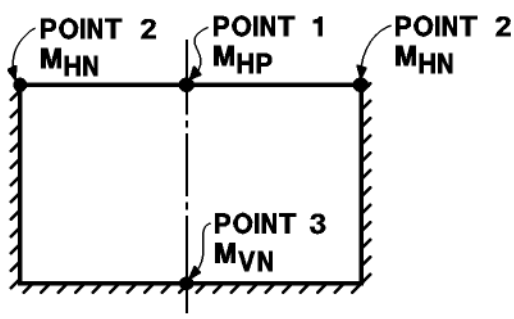
b. Considering a 1-inch strip (b = 1 inch)

Assume $I = 144$ in^4

Step 4. For points of interest, see Figure 3A-6 (+).

Step 5. Properties at first yield.

Figure 3A-6



From Figure 3-27 (+) for H/L = 0.7

$ß_1 = 0.077$ $ß_2 = 0.160$ $ß_3 = 0.115$

$\gamma_1 = 0.012$ $\nu = 0.3$

a. $M_{HP} = M_{HN} = M_{VP} = M_{VN}$ = 20,000 in-lbs/in

$M_P = ßrH^2$

$r = M/ßH^2$

r_1 = 20,000/[(0.077)(168)^2] = 9.20 psi

r_2 = 20,000/[(0.160)(168)^2] = 4.43 psi

r_3 = 20,000/[(0.115)(168)^2] = 6.16 psi

First yield at Point 2 (smallest r)

b. r_e = 4.43 psi

c. M_P (Point 1) = (0.077)(4.43)(168)^2 = 9,627 in-lbs/in

M_N (Point 3) = (0.115)(4.43)(168)^2 = 14,379 in-lbs/in

d. $D = EI/(b(1-\nu^2))$

= 29 x 10^6 x 144/(1(1-(0.3)^2)) = 45.9 x 10^8 in-lbs

$X_e = \gamma_1 r_e H^4/D$ = (0.0120)(4.43)(168)^4 /(45.9*10^8) = 0.0092 in

Step 6. Properties at second yield.

After first yield element assumes a simple-simple-fixed-free stiffness, therefore from Figure 3-29 (+) for H/L = 0.7:

$ß_1$ = 0.120 $ß_2$ = 0.220

γ_1 = 0.045 ν = 0.3

a. M_P (Point 1) = M_{HP} - M_P (at r_e) = 20,000 - 9627 = 10373 in-lbs/in

M_N (Point 3) = M_{VN} - M_P (at r_e) = 20,000 - 14,379 = 5621 in-lbs/in

b. M_P (Point 1) = 10373 in-lbs/in = $ß_1 \Delta r H^2$

Δr = 10373/(0.120)(168)^2 = 3.06 psi

M_n (Point 3) = 5,621 in-lbs/in = $ß_2 \Delta r H^2$

Δr = 5,621/[(0.220)(168)^2] = 0.90 psi

Second yield at Point 3 (smaller Δr)

c. Δr = 0.90 psi

d. $r_{ep} = r_e + \Delta r$ = 4.43 + 0.90 = 5.33 psi

e. M_P (Point 1) = 0.120(0.90)(168)^2 = 3.048 in-lbs/in

f. $D = EI/b(1-n^2)$ = (29)(10^6)(144)/1[1-(0.3)^2] = 45.9 x 10^8 in-lbs/in

$\Delta x = \gamma_1 \Delta r H^4/D$ = 0.030(0.90)(168)^4/45.9(10)^8 = 0.0047 in

g. $X_{ep} = X_e + \Delta X$ = 0.0092 + 0.0047 = 0.014 in

Step 7. Properties at final yield (ultimate unit resistance). After second yield element assumes a simple-simple-simple-free stiffness, therefore from Figure 3-30 (+) for H/L = 0.7:

$\gamma_1 = 0.041 \qquad \nu = 0.3$

a. r_u = 10.6 psi (Part (a), Example 3A-1(A))

b. $\Delta r = r_u - r_{ep} = 10.6 - 5.33 = 5.23$ psi

c. $D = EI/(b(1-\nu^2)) = 29(10^6)(144)/(1(1-(0.3)^2)) = 45.9 \times 10^8$ in-lbs

$\Delta x = \gamma_1 r H^4 / D = (0.045)(5.27)(168)^4/45.9 \times 10^8 = 0.037$ in

d. $X_p = X_{ep} + \Delta X = 0.014 + 0.041 = 0.051$ in

Step 8. For actual resistance-deflection curve, see Figure 3A-7 (+).

Step 9. $X_E = X_e(r_{ep}/r_u) + X_{ep}[1-(r_e/r_u)] + X_p[1-(r_{ep}/r_u)]$ Equation 3-35

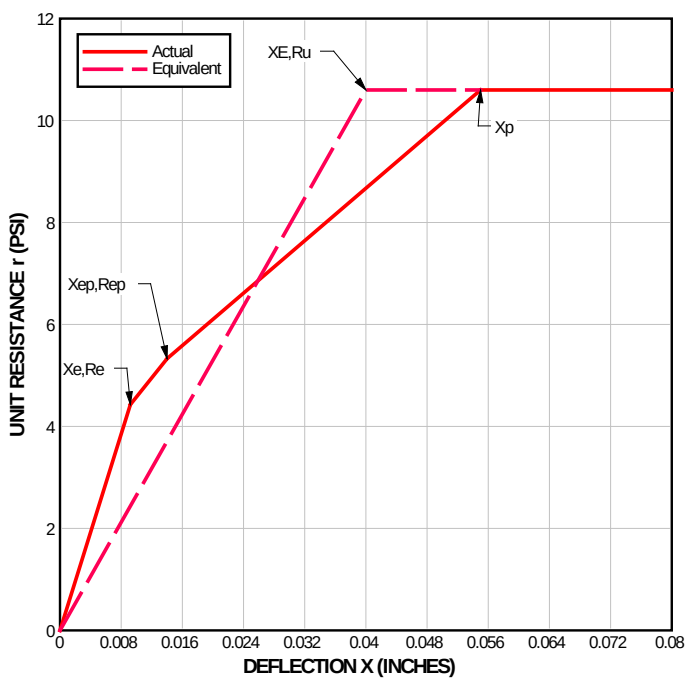
$X_E = 0.0092(5.37/10.6) + 0.014\,[1-(4.43/10.6)] + 0.051[1-(5.37/10.6)]$

$= 0.00466 + 0.0082 + 0.0252$

$= 0.038$ in

The equivalent resistance-deflection curve is shown in Figure 3A-7 (+).

Figure 3A-7



PROBLEM 3A-3 DYNAMIC DESIGN FACTORS FOR A ONE-WAY ELEMENT

Problem: **Determine the plastic load, mass and load-mass factors for a one-way element.**

Procedure:

Step 1. Establish design parameters.

Step 2. Determine deflected shape:

a. geometry of element

b. support conditions

c. type of load and mass

Step 3. Determine maximum deflection.

Step 4. Determine deflection function.

a. For distributed load and/or continuous mass determine the deflection at any point

b. For concentrated loads and concentrated mass determine the deflection at the load

Step 5. Calculate the shape function.

a. For distributed load and/or continuous mass calculate $\phi(x)$, Equation 3-43

b. For concentrated load and concentrated mass calculate ϕ_r, Equation 3-46

Step 6. Calculate the load factor, K_L.

a. Use Equations 3-41 and 3-42 for a distributed load

b. Use Equations 3-41 and 3-45 for a concentrated load

Step 7. Calculate the mass factor, K_M.

a. Use Equations 3-47 and 3-48 for a continuous mass

b. Use Equations 3-44 and 3-49 for concentrated mass

Step 8. Calculate the load-mass factor K_{LM}, from Equation 3-53.

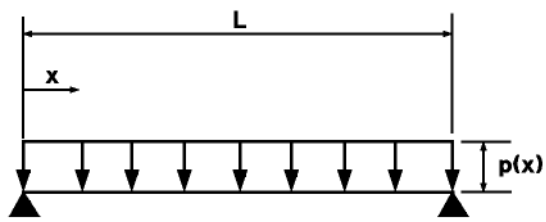
EXAMPLE 3A-3 (A)DYNAMIC DESIGN FACTORS FOR A ONE-WAY ELEMENT

Required: **The load, mass and load-mass factors for a structural steel beam in the elastic range, with a distributed load.**

Solution:

Step 1: Given structural steel beam shown in Figure 3A-8 (+)

Figure 3A-8



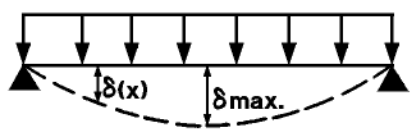
a. L = 120 in

b. Simply supported on both edges

c. $p(x)$ = 2,000 lb/in

$m(x)$ = 0.0055(lb-s^2/in)/in

Figure 3A-9



Step 2: Assumed deflected shape for elastic range is shown in Figure 3A-9 (+)

Step 3: The maximum deflection at the center is;

$$\delta(\max) = \frac{5p(\max)L^4}{384EI}$$

Step 4: Determine deflection function

$$\delta(x) = \frac{p(x)}{24EI}\left(L^3 - 2Lx^2 + x^3\right)$$

Step 5: Calculate the shape function using Equation 3-43

$$\phi = \frac{\delta(x)}{\delta_{\max}} = \frac{p(x)x}{24EI}\left(L^3 - 2Lx^2 + x^3\right)\left(\frac{384EI}{5p(x)L^4}\right)$$
$$= \frac{16}{5L^4}\left(L^3x - 2Lx^3 + x^4\right)$$

Step 6: a. Using Equation 3-42, determine equivalent force

$$F_E = \int_0^L p(x)\phi(x)dx = \int_0^{120}(2{,}000\,\text{lb/in})\frac{16}{5L^4}\left(L^3x - 2Lx^3 + x^4\right)dx$$
$$= \frac{6{,}400}{L^4}\left[\frac{L^3x^2}{2} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]_0^{120} = 1{,}280L$$
$$= 153{,}600\,\text{lb}$$

b. From Equation 3-41, find the load factor

$$K_L = \frac{F_E}{F} = \frac{153{,}600\,\text{lb}}{(2{,}000\,\text{lb/in} \times 120\,\text{in})}$$

K_L = 0.64 in the elastic range

Step 7: a. Find the equivalent mass from Equation 3-48:

$$M_E = \int_0^L m(x)\varphi^2(x)dx = 0.0055\left(\frac{256}{25L^8}\right)\left(\int_0^{120}\left(L^3x - 2Lx^3 + x^4\right)^2 dx\right)$$
$$= \frac{1.408}{25L^8}\left(\int_0^{120}\left(L^6x^2 - 4L^4x^4 + 2L^3x^5 + 4L^2x^6 - 4Lx^7 + x^8\right)dx\right)$$
$$= \frac{1.408}{25L^8}\left[\frac{L^6x^3}{3} - \frac{4L^4x^5}{5} + \frac{2L^3x^6}{6} + \frac{4L^2x^7}{7} - \frac{4Lx^8}{8} + \frac{x^9}{9}\right]_0^{120}$$
$$= 0.00277L$$
$$= 0.3325\ \text{lb} - \text{s}^2/\text{in}$$

b. From Equation 3-47, calculate the mass factor:

$$K_M = \frac{M_E}{M} = \frac{0.3325\,\text{lb} - s^2/\text{in}}{\left(0.0055\ \text{lb} - s^2/\text{in}^2 \times 120\,\text{in}\right)}$$

K_M = 0.50 in the elastic range

Step 8: Calculate the load-mass factor as defined by Equation 3-51:

$K_{LM} = K_M/K_L$

$= 0.50/0.64$

$K_{LM} = 0.78$ in the elastic range

EXAMPLE 3A-3 (B)DYNAMIC DESIGN FACTORS FOR A ONE-WAY ELEMENT

Required: **The load, mass and load-mass factors for a structural steel beam in the plastic range with a distributed load.**

Solution:

Step 1. Given the structural steel beam shown in Figure 3A-8 (+):

a. L = 120 in

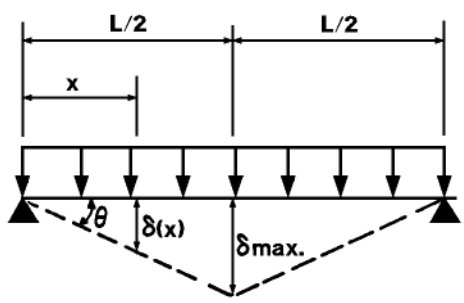
b. Simply-supported on both ends

c. $p(x)$ = 2,000 lb/in

d. $m(x)$ = 0.0055 (lb - s^2/in)/in

Step 2. Assume deflected shape for the plastic range is shown in Figure 3A-10 (+).

Figure 3A-10



Step 3. Determine maximum deflection:

$$\delta_{max} = (L/2)\tan\Theta$$

Step 4. Determine the deflection at any point:

$$\delta(x) = x\tan\Theta \qquad x < L/2$$

Step 5. Calculate the shape function, Equation 3-43:

$$\phi(x) = \frac{\delta(x)}{\delta_{max}} = \frac{x\tan\Theta}{(L/2)\tan\Theta}$$
$$= 2x/L \qquad x < L/2$$

Step 6:

a. Find F_E using Equation 3-42:

$$F_E = \int_0^L p(x)\phi(x)dx = 2\int_0^{60} (2{,}000\ \text{lb/in})(2x/L)dx$$
$$= 4{,}000\ \text{lb/in}\left[x^2/L\right]_0^{60}$$
$$= 120{,}000\ \text{lb}$$

b. From Equation 3-41:

$$K_L = \frac{F_E}{F} = \frac{120{,}000\ \text{lb}}{(2{,}000\ \text{lb/in})(120\ \text{in})}$$

K_L = 0.5 in the plastic range

Step 7:

a. Use Equation 3-48 to find the equivalent mass:

$$M_E = \int_0^L m(x)\varphi^2(x)dx = 2\int_0^{60} (0.0055)(4x^2/L^2)dx$$
$$= 0.044\left[\frac{x^3}{3L^2}\right]_0^{60}$$
$$= 0.22\ \text{lb}-s^2/\text{in}$$

b. As defined by Equation 3-47

$$K_M = \frac{M_E}{M} = \frac{0.22\ \text{lb}-s^2/\text{in}}{(0.0055\ \text{lb}-s^2/\text{in}^2)(120\ \text{in})}$$

K_M = 0.33 in the plastic range

Step 8. Calculate K_{LM} using Equation 3-53

$K_{LM} = K_M/K_L$

= 0.33/0.5

K_{LM} = 0.66 in the plastic range

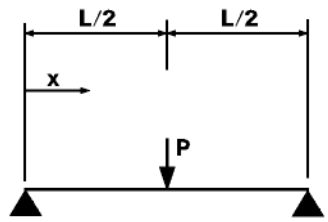
EXAMPLE 3A-3 (C)DYNAMIC DESIGN FACTORS FOR A ONE-WAY ELEMENT

Required: **The load, mass and load-mass factors for a structural steel beam in the elastic range, with a concentrated load.**

Solution:

Step 1: Given structural steel beam shown in Figure 3A-11 (+):

Figure 3A-11



a. L = 120 in

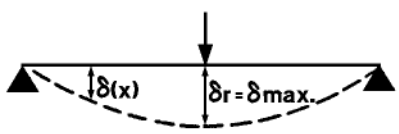
b. Simply supported on both sides

c. F = 240 kips

$m(x)$ = 0.0055(lb - s^2/in)/in

Step 2: Assume deflected shape for elastic range is shown in Figure 3A-12 (+):

Figure 3A-12



Step 3: Determine maximum deflection:

$$\delta_{\max} = \frac{PL^3}{48EI}$$

Step 4: Determine deflection functions:

a. for continuous mass

$$\delta(x) = \frac{Px}{48EI}\left(3L^2 - 4x^2\right)$$

b. for concentrated load

$$\delta_r = \frac{PL^3}{48EI}$$

Step 5: Calculate shape functions

a. For continuous mass use Equation 3-43

$$\phi(x) = \frac{\delta(x)}{\delta_{\max}} = \left(\frac{Px\left(3L^2 - 4x^2\right)}{48EI}\right)\left(\frac{48EI}{PL^3}\right)$$
$$= \left(3L^2x - 4x^3\right)/L^3$$

b. For concentrated load, use Equation 3-46

$$\phi_r = \left(\frac{PL^3}{48EI}\right)\left(\frac{48EI}{PL^3}\right)$$
$$= 1.0$$

Step 6: a. Find equivalent force from Equation 3-45

$$F_E = \sum^{i} F_r \phi_r = PxI = 240\text{kips}$$

b. Using Equation 3-41, calculate the load factor

$$K_L = \frac{F_E}{F} = \frac{240\text{kips}}{240\text{kips}}$$

K_L = 1.0 for the elastic range

Step 7: a. Equation 3-48 gives the equivalent mass.

$$M_E = \int_0^L m(x)\varphi^2(x)dx = 0.0055 \cdot \frac{256}{25L^8}\int_0^{120}\left(L^3x - 2Lx^3 + x^4\right)^2 dx$$
$$= 0.0027L$$
$$= 0.32\ \text{lb}\cdot s^2/\text{in}$$

b. From Equation 3-47, calculate the mass factor

$$K_M = \frac{M_E}{M} = \frac{0.32\ \text{lb}-s^2/\text{in}}{\left(0.0055\ \text{lb}-s^2/\text{in}^2 \times 120\ \text{in}\right)}$$

K_M = 0.49 in the elastic range

Step 8: Calculate the load-mass factor, from Equation 3-53

$K_{LM} = K_M/K_L$

= 0.49/1.0

K_{LM} = 0.49 for the elastic range

EXAMPLE 3A-3 (D)DYNAMIC DESIGN FACTORS FOR A ONE-WAY ELEMENT

Required: **Determine the load, mass and the load-mass factors for a structural steel beam, in the plastic range, with a concentrated load.**

Solution:

Step 1. Given structural steel beams shown in Figure 3A-11 (+).

a. L = 120 in

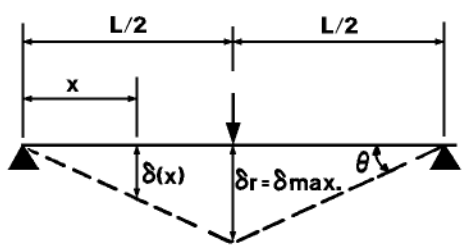
b. Simply-supported at both edges

c. F = 240 kips

$m(x)$ = 0.0055 (lb - s^2/in)/in

Step 2. Assumed deflected shape for the plastic range is shown in Figure 3A-13 (+)

Figure 3A-13



Step 3: Determine maximum deflection

δ_{max} = (L/2) tanΘ

Step 4: Determine deflection function.

a. for continuous mass

$\delta(x) = x \tan\Theta$ x < L/2

b. for a concentrated load

δ_r = (L/2) tanΘ

Step 5: Calculate shape factors using

a. Equation 3-43 for continuous mass

$$\phi(x) = \frac{\delta(x)}{\delta_{max}} = \frac{x \tan\Theta}{(L/2)\tan\phi}$$
$$= 2x/L \qquad x < L/2$$

b. Equation 3-46 for concentrated load

$$\phi_r = \frac{\delta_r}{\delta_{\max}} = \frac{(L/2)\tan\Theta}{(L/2)\tan\Theta}$$
$$= 1.0$$

Step 6: a. The equivalent force is found using Equation 3-45

$$F_E = \sum_{r=1}^{i} F_r \phi_r = Px_1 = 240\text{kips}$$

b. Equation 3-41 gives the load factor

$$K_L = \frac{F_E}{F} = \frac{240\text{kips}}{240\text{kips}}$$

K_L = 1.0 for plastic range

Step 7: a. The equivalent mass is found using Equation 3-48

$$M_E = \int_0^L m(x)\varphi^2(x)dx = 2\int_0^{60} (0.0055)(4x^2/L^2)dx$$
$$= 0.044\left[\frac{x^3}{3L^2}\right]_0^{60} = 0.22\text{lb}-s^2/\text{in}$$

b. Solve for K_M using Equation 3-47

$$K_M = \frac{M_E}{M} = \frac{0.22\ \text{lb}-s^2/\text{in}}{(0.0055\ \text{lb}-s^2/\text{in}^2)(120\ \text{in})}$$

K_M = 0.33 in the plastic range

Step 8: From Equation 3-53, calculate K_{LM}

$K_{LM} = K_M/K_L$

= 0.33 in the plastic range

PROBLEM 3A-4 PLASTIC LOAD-MASS FACTOR

Problem: **Determine the plastic load-mass factor *KLM* for a two-way element using (1) general solution and (2) chart solution.**

Note: The determination of the plastic load-mass factor follows the calculations for the ultimate resistance, hence the structural configuration and the location of the plastic yield lines will be known.

Procedure: **Part (a) - General Solution**

Step 1. See Part (a), Problem 3A-1 for the structural configuration and location of plastic yield lines. Denote sectors formed by yield lines.

Step 2. Determine the load-mass factors properties I, c, and L' for all sectors.

Step 3. Determine the factor I/cL' for all sectors.

Step 4. Calculate the total area of the element.

Step 5. With values obtained above, calculate the plastic load-mass factor for the element using Equation 3-57.

Note: In the above problem, an element of uniform thickness was considered. For non-uniform elements, the load-mass factor is calculated using Equation 3-53 where the mass of the individual sectors must be considered.

Procedure: **Part (b) - Chart Solution**

Step 1. See Part (b), Problem 3A-1 for structural configuration and location of plastic yield lines in terms of x/L or y/H.

Step 2. For known value of X/L or y/H and support condition, determine the load-mass factor for the element from Figure 3-44 (+).

Note: Chart solution may be used only if the element conforms to the requirements listed in Section 3-17.3.

EXAMPLE 3A-4 PLASTIC LOAD-MASS FACTOR

Required: **Plastic load-mass factor for the element considered in Example 3A-1(A) using (1) general solution and (2) chart solution.**

Solution: **Part (a) - General Solution**

Step 1. Given structural configuration and location of yield lines shown below (see Part (a), Example 3A-1(A)) in Figure 3A-14 (+).

L = 240 in

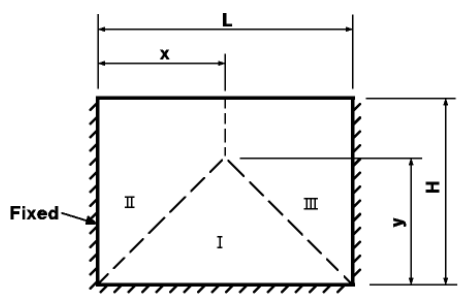
H = 168 in

X = 120 in

y = 137.6 in

T_c = constant

Figure 3A-14



Step 2. Load-mass factor properties.

a. Sector I.

$L' = y$ = 137.6 in

$c = y/3$ = 137.6/3

$I = L(L'^3)/12$ = 240(137.6)3/12.

b. Sector II.

Figure 3A-15

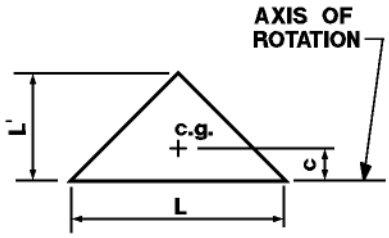
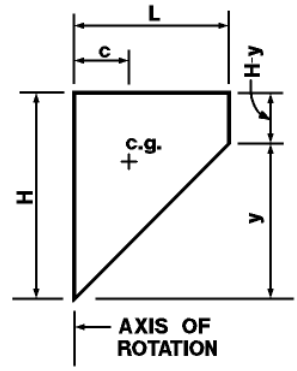


Figure 3A-16



$L' = x = 120$ in

$H - y = 168 - 137.6 = 30.4$ in

$$c = \frac{L'[H + 2(H - y)]}{3[H + (H - y)]} = \frac{120[168 + 2(30.4)]}{3(168 + 30.4)}$$

$c = 120\ (0.384)$

$$I = \frac{(H - y)(L')^3}{3} + \frac{y(L')^3}{12} = \frac{30.4(120)^3}{3} + \frac{137.6(120)^3}{12} = 21.60(120)^3$$

Step 3. Calculate factor I/cL' f or each sector:

Sector I. $$\frac{I}{cL'} = \frac{240(137.6)^3/12}{(133.4/3)(133.4)} = 8{,}784\text{in}^2$$

Sector II. $$\frac{I}{cL'} = \frac{21.60(120)^3}{(0.390 \times 120)(120)} = 6{,}646\text{in}^2$$

Sector III. $$\frac{I}{cL'} = 6{,}646\text{in}^2$$

Step 4. Area of panel

$A = LH = 240\ (168) = 40{,}320$ in^2

Step 5. Load-mass factor

$$K_{LM} = \frac{\sum I/cL'}{A}$$

$$= \frac{8{,}256 + 2(6{,}646)}{40{,}320} = 0.534$$

Solution: **Part (b) - Chart Solution**

Step 1. Given: Panel fixed on 3 edges, 1 free and y/H = 0.803 (see Part (b), Example 3A-1(A)).

Step 2. From Figure 3-44 (+), read load-mass factor

K_{LM} = 0.543

PROBLEM 3A-5 RESPONSE OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM SUBJECT TO DYNAMIC LOAD

Problem: **Determine the maximum response and the corresponding time it occurs of a single-degree-of-freedom system subjected to dynamic load using (a) numerical methods and (b) design charts.**

Procedure: **Part (a) - Numerical Methods**

Step 1. Establish dimensional parameters of the system.

Step 2. Determine the natural period of vibration and integration time interval.

Step 3. Construct a table similar to Table 3-14 (+) of Section 3-19.2.

Note: For the first interval n=1, Equation 3-59 is used and subsequent intervals, the recurrence formula (Equation 3-56) is used.

Procedure: **Part (b) - Chart solution**

Step 1. Same as Step 1 of Example 3A-5, Part (a).

Step 2. Determine the non-dimensional parameters.

Step 3. Determine the ratio of the maximum displacement to the elastic displacement X_m/X_E and the ratio of the time at which this maximum displacement occurs to the duration of the blast load.

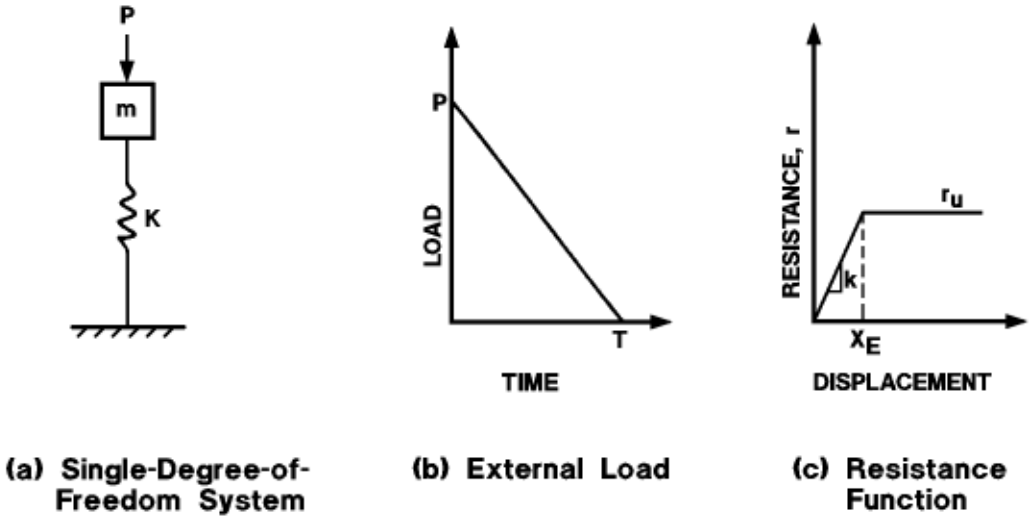
EXAMPLE 3A-5 MAXIMUM RESPONSE OF SINGLE-DEGREE-OF-FREEDOM SYSTEM SUBJECTED TO A TRIANGULAR LOAD.

Required: **The maximum response and the time it occurs, of a single-degree-of-freedom system subjected to blast loads, using (a) numerical methods and (b) design charts.**

Solution: **Part (a) - Numerical Methods**

Step 1. Given:

Figure 3A-17



m = 2.5 Kips-sec^2/ft

K = 9,860 Kips/ft

r_u = 750 Kips

X_E = 0.076 ft

T = 0.10 sec

P = 1000 Kips

Step 2. Natural period of vibration and integration time interval:

$T_N = 2\pi\,[m/K]^{1/2} = 2\pi\,[2.5/9{,}860]^{1/2} = 0.10$ sec

$t = T_N/10 = 0.01$ sec

Step 3. Construct table as shown below.

1	2	3	4	5	6	7	8	9	10
n	t	P_n	R_n	P_n-R_n	A_n=(P_n-R_n)/m	$a_n(\Delta t)^2$	$2X_n$	X_{n-1}	X_{n+1}
	(sec)	(Kips)	(Kips)	(Kips)	(ft/sec^2)	(ft)	(ft)	(ft)	(ft)
0	0	1000	0	1000	400	0.040	0.0	0.0	0.020
1	0.01	900	197.20	702.80	281.120	0.028112	0.0400	0.0	0.068112
2	0.02	800	671.68	128.43	51.366	0.05137	0.13622	0.020	0.121357
3	0.03	700	750	-50.0	-20.0	-0.00200	0.242714	0.06811	0.17261
4	0.04	600	750	-150.0	-60.0	-0.00600	0.34522	0.121357	0.21786
5	0.05	500	750	-250.0	-100.0	-0.0100	0.43673	0.17261	0.25312
6	0.06	400	750	-350.0	-140.0	-0.0140	0.050623	0.21786	0.27437
7	0.07	300	750	-450.0	-180.0	-0.0180	0.54874	0.25312	0.27762
8	0.08	200	750	-550.0	-220.0	-0.0220	0.55525	0.27437	0.25880
9	0.09								

Note: $P_n = f(t_n) = 1000\,[1 - n(\Delta t/T)]$

$$R_n = \begin{cases} KX_n \text{ for } X_n < X_e \\ r_u = 750 \text{ for } X_n > X_e \end{cases}$$

$X_{n+1} = 2X_n - X_{n-1} + a_n\,(\Delta t)^2$

Note:

For n=0, X_{n+1} (Column 10) $= X_{0+1} = X_1 = (1/2)a_0(\Delta t)^2$

$= (1/2)(0.040)$

$= 0.02$ ft

For n=1, $2X_n$ (Column 8) $= 2X_1 = 2[(1/2)a_0(\Delta t)^2]$

$= 2(0.02) = 0.04$ ft

X_{n-1} (Column 9) = $X_0 = 0.0$

X_{n+1} (Column 10) = $X_2 = 2X_1 - X_0 + a_1(\Delta t)^2$

$= 2(0.02) - 0 + 0.02811$

$= 0.06811$ ft

For n=2, $2X_n$ (Column 8) $= 2X_2 = 2(0.06811)$

$= 0.13622$ ft

X_{n-1} (Column 9) = $X_1 = 0.02$ ft.

X_{n+1} (Column 10) = $X_3 = 2X_2 - X_1 + a_2 (\Delta t)^2$

$= (2)(0.06811) - 0.02 + 0.005137$

$= 0.121357$ ft.

For n=3, 4,..., repeat the above procedure.

Solution: **Part (b) - Design Charts**

Step 1. Same as Step 1 of Example 3A-5, Part (a).

Step 2. Non-dimensional parameters:

a. Natural period of vibration, T_n

$T_n = 2\pi[m/K]^{1/2} = 2\pi[2.5/9{,}860]^{1/2} = 0.10$ sec

b. Ratio of duration of blast load T to natural period T_N

$T/T_N = 0.10/0.10 = 1.0$

c. Ratio of peak resistance r_u to peak load P

$r_u/P = 750/1000 = 0.75$

Step 3. Using the ratios calculated in Step 2 and Figures 3-54 (+) and 3-55 (+), determine the value of X_m/X_E and t_m/T_N.

For $T/T_N = 1$ and $r_u/P = 0.75$

$X_m/X_E = 3.7$ from Figure 3-54

$t_m/T = 0.77$ from Figure 3-55

Step 4. Determine X_m and t_m.

$X_m/X_E = 3.7$

$X_m = 3.7X_E = 3.7(.076) = 0.28144$ ft

$t_m/T = 0.77$

$t_m = 0.77T = 0.77(0.10) = 0.077$ sec

PROBLEM 3A-6 MAXIMUM RESPONSE OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM TO BILINEAR BLAST LOADS

Problem: **Determine X_m/X_E, t_m/T_N and t_E/T (when applicable) for a single-degree-of-freedom system subject to various bilinear blast loads.**

Procedure: **Part (a) - Solution in Region D**

Step 1. Establish normalized parameters.

Step 2. Enter Table 3-15 (+) with the given C parameters and determine which figures have to be used.

Step 3. Enter each of the figures determined in Step 2, with the given values of the other two parameters and determine the region where the intersection points are located.

Step 4. Based on the region where the intersection points are located, enter the appropriate figure and find X_m/X_E, t_m/T_N and t_E/T.

Procedure: **Part (b) - Solution in Region C - Graphical Interpolation.**

Step 1. Same as Step 1 in Part (a).

Step 2. Same as Step 2 in Part (a).

Step 3. Same as Step 3 in Part (a).

Step 4. Set up a table as shown in Table 3A-1 (+). Post each figure number and the corresponding values of C_1 and C_2, leaving a space between each line of information. Post in the spaces the appropriate values of C_1 and C_2 needed for interpolation. Enter each of the figures determined in Step 2 with the given parameters and find the values of X_m/X_E. Post these values in Table 3A-1.

Step 5. Use log-log graph paper to plot the points obtained in Step 4. Post these values in Table 3A-1 (+), using linear interpolation where necessary.

Step 6. Plot on log-log graph paper the points which represent (X_m/X_E, C_2) for the given value of C. Use linear interpolation to find X_m/X_E for given value of C_2.

Procedure: **Part (c) - Solution in Region C - Mathematical Interpolation**

Step 1. Same as Step 1 in Part (a).

Step 2. Same as Step 2 in Part (a).

Step 3. Same as Step 3 in Part (a).

Step 4. Same as Step 4 in Part (b).

Step 5. Solve for $\ln Y_a$ and $\ln Y_b$ using Equations 3-83 and 3-84.

Step 6. Solve for $\ln Y$ using Equation 3-85.

Step 7. Solve for Y using Equation 3-86.

EXAMPLE 3A-6 MAXIMUM RESPONSE OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM TO BILINEAR BLAST LOADS

Required: **Determine X_m/X_E, t_m/T_N and t_E/T (when applicable) for a single-degree-of-freedom system subject to various bilinear blast loads.**

Solution: **Part (a) - Solution in Region D**

Step 1. Given: $P/r_u = 1.0$

$T/T_N = 3.0$

$C_1 = 0.66$

$C_2 = 50$

Step 2. Enter Table 3-15 (+) with $C_1 = 0.66$ and $C_2 = 50$. Note Figures 3-119 (+/+/+), 3-120 (+/+/+), 3-147 (+/+/+), and 3-148 (+/+/+) apply.

Step 3. Enter each of the figures determined in Step 2, with $P/r_u = 1.0$ and $T/T_N = 3.0$. Note that the intersection point is located to the right of the line of solid squares, defined as Region D. In Region D, the maximum dynamic response depends only on the shock load described by P/r_u and T/T_N; the gas load described by C_1P/r_u and C_2T/T_n does not influence the maximum dynamic response. Consequently, Figures 3-64a (+) and 3-64b (+) for a single triangular load pulse apply. Enter Figure 3-64a with $P/r_u = 1.0$ and $T/T_N = 3.0$ and find $X_m/X_E = 3.55$, $t_m/T_N = 0.98$, $t_E/T = 0.086$.

Solution: **Part (b) - Solution in Region C - Graphical Interpolation**

Step 1. Given: $P/r_u = 32$

$T/T_N = 0.10$

$C_1 = 0.06$

$C_2 = 20$

Step 2. Enter Table 3-15 (+) with $C_1 = 0.06$ and $C_2 = 20$. Note Figures 3-108 (+/+/+), 3-109 (+/+/+), 3-133 (+/+/+) and 3-134 (+/+/+) apply.

Step 3. Enter each of these Figures with $P/r_u = 32$ and $T/T_n = 0.10$. Note that the intersection point is not located in Regions A, B or D. Therefore the intersection points lie in region C and interpolation between charts is required to obtain a solution.

Step 4. Set up table as shown in Table 3A-1 (+) below. Post each chart number and the corresponding values of C_1 and C_2 leaving a space between each line of information. Post in the spaces the appropriate values of C_1 and C_2 needed for interpolation. Enter Figure 3-108 (+/+/+) with $P/r_u = 32$ and

T/T_N = 0.10 and find X_m/X_E = 112. Post this value in the table. Enter Figure 3-109 (+/+/+) with P/r_u = 32 and T/T_N = 0.10 and find X_m/X_E = 86. Post this value in the table. Repeat this process for Figures 3-133 (+/+/+) and 3-134 (+/+/+), and post values for X_m/X_E in the table.

Step 5. Use log-log graph paper to plot the points (112, 0.068) and (86, 0.046) which represent (X_m/X_E, C_1) for C_2 = 10 as shown in Figure 3-267 (+). Use straight-line interpolation to find X_m/X_E = 103 for C_1 = 0.060. Post this value in the table. Repeat this process for C_2 = 30, and find X_m/X_E = 251 for C_1 = 0.06 as shown in Figure 3-267.

Step 6. Plot on log-log graph paper the points (103, 10) and (251, 30) which represent (X_m/X_E, C_2) for C_1, = 0.060. Use straight-line interpolation for finding X_m/X_E = 182 for C_2 = 50 as shown in Figure 3-267 (+). Thus the solution is X_m/X_E = 182.

Figure No.
3-108 (+/+/+)
3-109 (+/+/+)
3-133 (+/+/+)
3-134 (+/+/+)

Solution: Part (c) - Solution in Region C - Mathematical Interpolation

Step 1. Same as Step 1 of Part (b).

Step 2. Same as Step 2 of Part (b).

Step 3. Same as Step 3 of Part (b).

Step 4. Same as Step 4 of Part (b).

Step 5. Using Equations 3-83 and 3-84, find $\ln Y_a$ and $\ln Y_b$.

$$\ln Y_a = \ln Y_1 + \frac{\ln[Y_2 / Y_1]\ln[C_1 / C_{11}]}{\ln[C_{12} / C_{11}]}$$

$$= \ln(112) + \frac{\ln(86/112)\ln(0.060/0.068)}{\ln(0.046/0.068)}$$

$$\ln Y_a = 4.6339$$

$$\ln Y_b = \ln Y_3 + \frac{\ln[Y_4 / Y_3]\ln[C_1 / C_{13}]}{\ln[C_{14} / C_{13}]}$$

$$= \ln(340) + \frac{\ln(230/340)\ln(0.06/0.075)}{\ln(0.056/0.075)}$$

$$\ln Y_b = 5.5304$$

Step 6. Find ln$Y$ from Equation 3-85:

$$\ln Y = \ln Y_a + \frac{(\ln Y_b - \ln Y_a)\ln(C_2 / C_{21})}{\ln(C_{23} / C_{21})}$$

$$= 4.6339 + \frac{(5.5304 - 4.639)\ln(20/10)}{\ln(30/10)}$$

$$\ln Y = 5.1995$$

Step 7. Solve for Y using Equation 3-86:

$$Y = \mathrm{e}^{\ln Y}$$

$$= e^{5.1995}$$

$$Y = 181$$

APPENDIX 3B LIST OF SYMBOLS

a	(1)	acceleration (in/ms^2)
	(2)	depth of equivalent rectangular stress block (in)
A		area (in^2)
A_a		area of diagonal bars at the support within a width b (in^2)
A_o		area of openings (ft^2)
A_s		area of tension reinforcement within a width b (in^2)
A'_s		area of compression reinforcement within a width b (in^2)
A_{sH}		area of flexural reinforcement within a width b in the horizontal direction on each face (in^2)*
A_{sV}		area of flexural reinforcement within a width b in the vertical direction on each face (in^2)*
A_v		total area of stirrups or lacing reinforcement in tension within a distance, s_s or s_l and a width b_s or b_l (in^2)
A_I, A_{II}		area of sector I and II, respectively (in^2)
b	(1)	width of compression face of flexural member (in)
	(2)	width of concrete strip in which the direct shear stresses at the supports are resisted by diagonal bars (in)
b_s		width of concrete strip in which the diagonal tension stresses are resisted by stirrups of area A_v (in)
b_1		width of concrete strip in which the diagonal tension stresses are resisted by lacing of area A_v (in)
B		constant defined in paragraph
c	(1)	distance from the resultant applied load to the axis of rotation (in)
	(2)	damping coefficient
c_I, c_{II}		distance from the resultant applied load to the axis of rotation for sectors I and II, respectively (in)
c_s		dilatational velocity of concrete (ft/sec)
C		shear coefficient
C_{cr}		critical damping

* See note at end of List of Symbols.

C_d	shear coefficient for ultimate shear stress of one-way elements
C_f	post-failure fragment coefficient (lb^2-ms^4/in^8)
$C_{r\alpha}$	peak reflected pressure coefficient at angle of incidence α
C_s	shear coefficient for ultimate support shear for one-way elements
C_{sH}	shear coefficient for ultimate support shear in horizontal direction for two-way elements*
C_{sV}	shear coefficient for ultimate support shear in vertical direction for two-way elements*
C_D	drag coefficient
$C_D q$	drag pressure (psi)
$C_D q_o$	peak drag pressure (psi)
C_E	equivalent load factor
C_H	shear coefficient for ultimate shear stress in horizontal direction for two-way elements*
C_L	leakage pressure coefficient
C_M	maximum shear coefficient
C_u	impulse coefficient at deflection X_u (psi-ms^2/in^2)
C_u'	impulse coefficient at deflection X_m (psi-ms^2/in^2)
C_v	shear coefficient for ultimate shear stress in vertical direction for two-way elements*
C_1	(1) impulse coefficient at deflection X_1 (psi-ms^2/in^2)
	(2) parameter defined in figure
	(3) ratio of gas load to shock load
C_1'	impulse coefficient at deflection X_m (psi-ms^2/in^2)
C_2	ratio of gas load duration to shock load duration
d	distance from extreme compression fiber to centroid of tension reinforcement (in)
d'	distance from extreme compression fiber to centroid of compression reinforcement (in)

* See note at end of List of Symbols.

d_c	distance between the centroids of the compression and tension reinforcement (in)
d_e	distance from support and equal to distance d or d_c (in)
d_i	inside diameter of cylindrical explosive container (in)
d_l	distance between center lines of adjacent lacing bends measured normal to flexural reinforcement (in)
d_{co}	diameter of steel core (in)
d_1	diameter of cylindrical portion of primary fragment (in)
D	(1) unit flexural rigidity (lb-in)
	(2) location of shock front for maximum stress (ft)
	(3) minimum magazine separation distance (ft)
D_o	nominal diameter of reinforcing bar (in)
D_E	equivalent loaded width of structure for non-planar wave front (ft)
DIF	dynamic increase factor
DLF	dynamic load factor
e	base of natural logarithms and equal to 2.71828...
$(2E')^{1/2}$	Gurney Energy Constant (ft/sec)
E	modulus of elasticity
E_c	modulus of elasticity of concrete (psi)
E_s	modulus of elasticity of reinforcement (psi)
f	unit external force (psi)
f'_c	static ultimate compressive strength of concrete at 28 days (psi)
f'_{dc}	dynamic ultimate compressive strength of concrete (psi)
f_{ds}	dynamic design stress for reinforcement (psi)
f_{du}	dynamic ultimate stress of reinforcement (psi)
f_{dy}	dynamic yield stress of reinforcement (psi)
f_s	static design stress for reinforcement (a function of f_y, f_u and Θ) (psi)
f_u	static ultimate stress of reinforcement (psi)
f_y	static yield stress of reinforcement (psi)

F	(1)	total external force (lbs)
	(2)	coefficient for moment of inertia of cracked section
	(3)	function of C_2 and C_1 for bilinear triangular load
F_o		force in the reinforcing bars (lbs)
F_E		equivalent external force (lbs)
g		variable defined in Table 4-3 (+)
h		charge location parameter (ft)
H	(1)	span height (in)
	(2)	distance between reflecting surface(s) and/or free edge(s) in vertical direction (ft)
H_c		height of charge above ground (ft)
H_c		scaled height of charge above ground (ft/lb$^{1/3}$)
H_s		height of structure (ft)
H_T		scaled height of triple point (ft/lb$^{1/3}$)
i		unit positive impulse (psi-ms)
i^-		unit negative impulse (psi-ms)
$\overline{i_a}$		sum of scaled unit blast impulse capacity of receiver panel and scaled unit blast impulse attenuated through concrete and sand in a composite element (psi-ms/lb$^{1/3}$)
i_b		unit blast impulse (psi-ms)
$\overline{i_b}$		scaled unit blast impulse (psi-ms/lb$^{1/3}$)
$\overline{i_{bt}}$		total scaled unit blast impulse capacity of composite element (psi-ms/lb$^{1/3}$)
$\overline{i_{ba}}$		scaled unit blast impulse capacity of receiver panel of composite element (psi-ms/lb$^{1/3}$)
$\overline{i_{bd}}$		scaled unit blast impulse capacity of donor panel of composite element (psi-ms/lb$^{1/3}$)
i_e		unit excess blast impulse (psi-ms)
i_r		unit positive normal reflected impulse (psi-ms)
i_r^-		unit negative normal reflected impulse (psi-ms)
i_s		unit positive incident impulse (psi-ms)

i_s^-	unit negative incident impulse (psi-ms)
I	moment of inertia (in^4)
I_a	average of gross and cracked moments of inertia of width b (in^4)
I_c	moment of inertia of cracked concrete section of width b (in^4)
I_g	moment of inertia of gross concrete section of width b (in^4)
I_m	mass moment of inertia (lb-ms^2-in)
j	ratio of distance between centroids of compression and tension forces to the depth d
k	constant defined in paragraph
K	(1) unit stiffness (psi-in for slabs) (lb/in/in for beams)
	(2) constant defined in paragraph
K_e	elastic unit stiffness (psi/in for slabs) (lb/in/in for beams)
K_{ep}	elasto-plastic unit stiffness (psi-in for slabs) (psi for beams)
K_E	equivalent elastic unit stiffness (psi-in for slabs) (psi for beams)
K_L	load factor
K_{LM}	load-mass factor
$(K_{LM})_u$	load-mass factor in the ultimate range
$(K_{LM})_{up}$	load-mass factor in the post-ultimate range
K_M	mass factor
K_R	resistance factor
K_1	factor defined in paragraph
KE	kinetic energy
l	charge location parameter (ft)
l_p	spacing of same type of lacing bar (in)
L	(1) span length (in) except in chapter 4 (ft)*
	(2) distance between reflecting surface(s) and/or free edge(s) in horizontal direction (ft)
L_l	length of lacing bar required in distance s_l (in)

* See note at end of List of Symbols.

L_o	embedment length of reinforcing bars (in)
L_w	wave length of positive pressure phase (ft)
L_w^-	wave length of negative pressure phase (ft)
L_{wb}, L_{wd}	wave length of positive pressure phase at points b and d, respectively (ft)
L_1	total length of sector of element normal to axis of rotation (in)
m	unit mass (psi-ms^2/in)
m_a	average of the effective elastic and plastic unit masses (psi-ms^2/in)
m_e	effective unit mass (psi-ms^2/in)
m_u	effective unit mass in the ultimate range (psi-ms^2/in)
m_{up}	effective unit mass in the post-ultimate range (psi-ms^2/in)
M	(1) unit bending moment (in-lbs/in)
	(2) total mass (lb-ms^2/in)
M_e	effective total mass (lb-ms^2/in)
M_u	ultimate unit resisting moment (in-lbs/in)
M_c	moment of concentrated loads about line of rotation of sector (in-lbs)
M_A	fragment distribution parameter
M_E	equivalent total mass (lb-ms^2/in)
M_{HN}	ultimate unit negative moment capacity in horizontal direction (in-lbs/in)*
M_{HP}	ultimate unit positive moment capacity in horizontal direction (in-lbs/in)*
M_N	ultimate unit negative moment capacity at supports (in-lbs/in)
M_P	ultimate unit positive moment capacity at midspan (in-lbs/in)
M_{VN}	ultimate unit negative moment capacity in vertical direction (in-lbs/in)*
M_{VP}	ultimate unit positive moment capacity in vertical direction (in-lbs/in)*
n	(1) modular ratio
	(2) number of time intervals
N	number of adjacent reflecting surfaces
N_f	number of primary fragments larger than W_f

* See note at end of List of Symbols.

p	reinforcement ratio equal to $\frac{A_s}{bd}$ or $\frac{A_s}{bd_c}$
p'	reinforcement ratio equal to $\frac{A_s'}{bd}$ or $\frac{A_s'}{bd_c}$
p_b	reinforcement ratio producing balanced conditions at ultimate strength
p_m	mean pressure in a partially vented chamber (psi)
p_{mo}	peak mean pressure in a partially vented chamber (psi)
p_H	reinforcement ratio in horizontal direction on each face*
p_T	reinforcement ratio equal to $p_H + p_v$
p_v	reinforcement ratio in vertical direction on each face*
$p(x)$	distributed load per unit length
P	(1) pressure (psi)
	(2) concentrated load (lbs)
P^-	negative pressure (psi)
P_i	interior pressure within structure (psi).
ΔP_i	interior pressure increment (psi)
P_f	fictitious peak pressure (psi)
P_o	peak pressure (psi)
P_r	peak positive normal reflected pressure (psi)
P_r^-	peak negative normal reflected pressure (psi)
$P_{r\alpha}$	peak reflected pressure at angle of incidence α (psi)
P_s	positive incident pressure (psi)
P_{sb}, P_{se}	positive incident pressure at points b and e, respectively (psi)
P_{so}	peak positive incident pressure (psi)
P_{so}^-	peak negative incident pressure
P_{sob}, P_{sod}, P_{soe}	peak positive incident pressure at points b, d, and e, respectively (psi)
q	dynamic pressure (psi)

* See note at end of List of Symbols.

q_b, q_e	dynamic pressure at points b and e, respectively (psi)
q_o	peak dynamic pressure (psi)
q_{ob}, q_{oe}	peak dynamic pressure at points b and e, respectively (psi)
r	(1) unit resistance (psi)
	(2) radius of spherical TNT (density equals 95 lb/ft^3 charge (ft))
r^-	unit rebound resistance (psi)
Δr	change in unit resistance (psi)
r_e	elastic unit resistance
r_{ep}	elasto-plastic unit resistance (psi)
r_u	ultimate unit resistance (psi, for slabs) (lb/in for beams)
r_{up}	post-ultimate unit resistant (psi)
r_1	radius of hemispherical portion of primary fragment (in)
R	(1) total internal resistance (lbs)
	(2) slant distance (ft)
R_f	distance traveled by primary fragment (ft)
R_l	radius of lacing bend (in)
R_A	normal distance (ft)
R_E	equivalent total internal resistance (lbs)
R_G	ground distance (ft)
R_u	total ultimate resistance
R_I, R_{II}	total internal resistance of sectors I and II, respectively (lbs)
s_s	spacing of stirrups in the direction parallel to the longitudinal reinforcement (in)
s_l	spacing of lacing in the direction parallel to the longitudinal reinforcement (in)
S	height of front wall or one-half its width, whichever is smaller (ft)
SE	strain energy
t	time (ms)
Δt	time increment (ms)
t_a	any time (ms)

t_b, t_e, t_f	time of arrival of blast wave at points b, e, and f, respectively (ms)
t_c	(1) clearing time for reflected pressures (ms)
	(2) container thickness of explosive charges (in)
t_d	rise time (ms)
t_E	time to reach maximum elastic deflection
t_m	time at which maximum deflection occurs (ms)
t_o	duration of positive phase of blast pressure (ms)
t_o^-	duration of negative phase of blast pressure (ms)
t_{of}	fictitious positive phase pressure duration (ms)
t_{of}^-	fictitious negative phase pressure duration (ms)
t_r	fictitious reflected pressure duration (ms)
t_u	time at which ultimate deflection occurs (ms)
t_y	time to reach yield (ms)
t_A	time of arrival of blast wave (ms)
t_1	time at which partial failure occurs (ms)
T	duration of equivalent triangular loading function (ms)
T_c	thickness of concrete section (in)
T_c	scaled thickness of concrete section (ft/lb$^{1/3}$)
T_N	effective natural period of vibration (ms)
T_r	rise time (ms)
T_s	thickness of sand fill (in)
T_s	scaled thickness of sand fill (ft/lb$^{1/3}$)
u	particle velocity (ft/ms)
u_u	ultimate flexural or anchorage bond stress (psi)
U	shock front velocity (ft/ms)
v	velocity (in/ms)
v_a	instantaneous velocity at any time (in/ms)
v_b	boundary velocity for primary fragments (ft/sec)
v_c	ultimate shear stress permitted on an unreinforced web (psi)

v_f	maximum post-failure fragment velocity (in/ms)
v_f (avg.)	average post-failure fragment velocity (in/ms)
v_i	velocity at incipient failure deflection (in/ms)
v_o	initial velocity of primary fragment (ft/sec)
v_r	residual velocity of primary fragment after perforation (ft/sec)
v_s	striking velocity of primary fragment (ft/sec)
v_u	ultimate shear stress (psi)
v_{uH}	ultimate shear stress at distance d_e from the horizontal support (psi)*
v_{uV}	ultimate shear stress at distance d_e from the vertical support (psi)*
V	volume of partially vented chamber (ft^3)
V_d	ultimate direct shear capacity of the concrete of width b (lbs)
V_{dH}	shear at distance d_e from the vertical support on a unit width (lbs/in)*
V_{dV}	shear at distance d_e from the horizontal support on a unit width (lbs/in)*
V_o	volume of structure (ft^3)
V_s	shear at the support on a unit width (lbs/in)*
V_{sH}	shear at the vertical support on a unit width (lbs/in)*
V_{sV}	shear at the horizontal support on a unit width (lbs/in)*
V_u	total shear on a width b (lbs)
w	weight density of concrete (lbs/ft^3)
w_s	weight density of sand (lbs/ft^3)
W	charge weight (lbs)
W_c	total weight of explosive containers (lbs)
W_f	weight of primary fragment (oz)
W_{co}	total weight of steel core (lbs)
W_{c1}, W_{c2}	total weight of plates 1 and 2, respectively (lbs)
W_s	width of structure (ft)
WD	work done
x	yield line location in horizontal direction (in) *

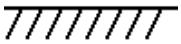
* See note at end of List of Symbols.

X	deflection (in)
X_a	any deflection (in)
X_e	elastic deflection (in)
X_{ep}	elasto-plastic deflection (in)
X_f	maximum penetration into concrete of armor-piercing fragments (in)
X_f'	maximum penetration into concrete of fragments other than armor-piercing (in)
X_m	maximum transient deflection (in)
X_p	plastic deflection (in)
X_s	(1) maximum penetration into sand of armor-piercing fragments (in)
	(2) static deflection (in)
X_u	ultimate deflection (in)
X_E	equivalent elastic deflection (in)
X_1	partial failure deflection (in)
y	yield line location in vertical direction (in)*
Z	scaled slant distance (ft/lb$^{1/3}$)
Z_A	scaled normal distance (ft/lb$^{1/3}$)
Z_G	scaled ground distance (ft/lb$^{1/3}$)
α	(1) angle formed by the plane of stirrups, lacing, or diagonal reinforcement and the plane of the longitudinal reinforcement (deg)
	(2) angle of incidence of the pressure front (deg)
$ß$	(1) coefficient for determining elastic and elasto-plastic resistances
	(2) particular support rotation angle (deg)
γ	(1) coefficient for determining elastic and elasto-plastic deflections
	(2) deflection's increase in support rotation angle after partial failure (deg)
Θ	(1) support rotation angle (deg)
	(2) angular acceleration (rad/ms^2)
Θ_{max}	maximum support rotation angle (deg)

Θ_H	horizontal rotation angle (deg) *
Θ_V	vertical rotation angle (deg)*
Σo	effective perimeter of reinforcing bars (in)
ΣM	summation of moments (in-lbs)
ΣM_N	sum of the ultimate unit resisting moments acting along the negative yield lines (in-lbs)
ΣM_P	sum of the ultimate unit resisting moments acting along the positive yield lines (in-lbs)
μ	ductility factor
ν	Poisson's ratio
ϕ	(1) capacity reduction factor
	(2) bar diameter (in)
ϕ_r	assumed shape function for concentrated loads
$\phi(x)$	assumed shape function for distributed loads

 free edge

simple support

 fixed support

either fixed, restrained, or simple support

* See note at end of List of Symbols.

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